Effect of Pulse Shaping on BER Performance of QAM Modulated OFDM Signal

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Abstract
Pulse shaping of a signal at base-band forms a more important tool in controlling and analyzing the parameters like ISI, the modulated signal envelope uniformity, the phase continuity as these parameters lead to the efficient spectral confinement of a binary signal which is a future and nowadays a major important aspect in the design of a wireless cellular communication system.

In this paper, different time-limited waveform are discussed which are basically available as windows functions and comparing the performance of Rectangular, Raised cosine and BTRC and prove that the BTRC pulse is supposed to be a better pulse shaping for OFDM in terms of obtaining better BER performance and proposed two pulse shapes which are giving better $E[\rho_i^2]$ in respective cases which in turn provides better BER as compared to BTRC which was considered as one of the best pulse shape.

Keywords: Inter Symbol Interference, Inter Channel Interference, Signal to Noise Ratio, Bit Error Rate.

Introduction
A linear filter channel distorts the transmitted signal and the parameter like ISI is an undesired overlap of the neighboring symbols both in time and in phase. The channel distortions results in ISI at the output of the demodulator which leads to an increase in ISI at the output of the demodulator and leads to an increase in the probability of error at the detector and hence BER performance is degraded. Greater ISI allows the spectrum to be more compact, making demodulator more complex, hence, spectral compactness is the primary trade-off in going from one type of pulse shaping to other type i.e. the pulse shaping can take care of the BER performance of the OFDM system and can be a major tool to reduce the ISI and ICI effects efficiently in an OFDM system [1, 2]. The pre-modulation low pass filter (LPF) must have a narrow Bandwidth with sharp cutoff frequency and very little overshoot in its impulse response.

In this paper, different time-limited waveform are discussed which are basically available as windows functions. The synthesis of window function is an important technique for pulse shaping [1], [3] and the pulse shaping gives a considerable performance improvement of OFDM systems in multipath radio channels as compared to the conventional OFDM. Therefore, windowing finds a wide and large area of applications extending from spectrum estimation, digital filtering, speech processing, surface acoustic wave (SAW) filter design [4], [5] and many other fields of communication and signal processing.

An exhaustive review of many pulses shaping functions or time limited waveform is done on the basis of their application and characteristics in [3] with a conclusion that the Comparing the performance of Rectangular, Raised cosine and BTRC, the BTRC gives smaller equivalent noise power and hence lower BER values. Therefore, the BTRC pulse is supposed to be a better pulse shaping for OFDM in terms of obtaining better BER performance [6].

There after a good number of windows have been proposed by various authors with some performing better in the frequency domain and some in the time domain. A concise comparative study has also been performed in different way [7]. The main criterion for the selection of a window is application oriented, means that whether the application requires better spectral performances or the better time domain performances [8] and it can be concluded that the two proposed pulses are giving better $E[\rho_i^2]$ in respective cases which in turn provides better BER as compared to BTRC which was considered as one of the best pulse shape [6].

ICI IN OFDM
The pulse shaping plays a major role in improvising the Bit Error Rate (BER) performance of modulated OFDM system [9], [7], since the BER is related to the signal to noise ratio (SNR) of a signal where signal power is supposed to be the second moment of the amplitude of the pulse shaping signal, where it can be observed that the role off factor ‘$\alpha$’ is present in the second moment of amplitude which indicates that the role off factor is responsible for the performance of BER of the signal. This indicates that a play in the role off factor for a particular pulse shaping can improve the BER performance.

Since, ICI power is the nothing but the mean square value of an ICI term present in an OFDM transmitted signal and is dependent on the amplitude or magnitude of the weighting function which is otherwise also known as pulse shaping function. Thus, it is evident that the play in the role off factor of a pulse shape will affect the ICI power also of the transmitted OFDM signal.
Thus, in order to analyze various pulse shapes in the light of BER performance firstly the amplitude moment of each of them has to be calculated, the ICI of an OFDM signal is calculated as per equation (1).

\[ \rho = \sum_{k=0}^{N-1} a[k] \cdot g(-kT-\tau) = \sum_{k=-\infty}^{\infty} a[k] \cdot C_{-m} \sum_{k=0}^{N-1} \alpha \cdot C_{-m} \]  

(1)

and the equivalent noise \( E[\rho^2] \) in geometrical progression (G. P.) is derived as-

\[ E[\rho^2] = \sum_{k=0}^{N-1} E\left[\left(a[k] g(-kT-\tau)\right)^2\right] \]  

(2)

\[ E[\rho^2] = \sum_{k=-\infty}^{\infty} E\left[C_{-m}^2\right] \]  

\[ E[\rho^2] = \sum_{-\infty}^{\infty} \int_{-\infty}^{\infty} g^2(-kT-\tau) d\tau = \sum_{-\infty}^{\infty} \int_{-\infty}^{\infty} g^2(t) dt \]

replacing the factor (-Kt - \( \tau \)) by \( t \) as-

\[ E[\rho^2] = \int_{-\infty}^{\infty} g^2(t) dt = \frac{1}{T} \int_{-T/2}^{T/2} \left| G(f) \right|^2 df \]

which is nothing but the Parseval’s Theorem.

**Types of Pulse Shapes in OFDM**

As lot of pulse shaping functions have been designed, optimized and applied for various applications, with a result some performing better in time domain whereas some in frequency domain. In this section some most commonly used pulse shaping functions which define a class of modulation techniques are included [3, 10-11, 12 & 13].

**Case-I: The Rectangular Pulse**

\[ G(f) = \frac{T}{\pi} \text{rect} \left( \frac{fT}{\pi} \right) \]

**Case-II: Raised Cosine (RC) Pulse**

\[ G(f) = T, \text{ for } 0 \leq |f| \leq \frac{(1-\alpha)}{2T} \]

\[ G(f) = \frac{T}{2} \left[ 1 + \cos \left( \frac{\pi T}{\alpha} \left| \left| f \right| - \frac{1-\alpha}{2T} \right| \right) \right], \text{ for } \frac{(1-\alpha)}{2T} \leq |f| \leq \frac{(1+\alpha)}{2T} \]

\[ G(f) = 0, \text{ otherwise} \]

(4)

(5)

where, the alpha (\( \alpha \)) is known as the pulse shaping factor lies \( 0 \leq \alpha \leq 1 \).

**Case-III: Better than Raised Cosine (BTRC) Pulse**

\[ G(f) = T, \text{ for } 0 \leq |f| \leq \frac{(1-\alpha)}{2T} \]

\[ G(f) = \frac{T}{2} e^{-\frac{2T \log_2 \left| \left| f \right| - \frac{1-\alpha}{2T} \right|}{\alpha}} \]  

\[ G(f) = \frac{T}{2} e^{-\frac{2T \log_2 \left| \left| f \right| - \frac{1+\alpha}{2T} \right|}{\alpha}} \]

\[ G(f) = 0, \text{ otherwise} \]

(6)

Thus, the BTRC pulse shape has a small BER in presence of ISI and symbol timing error in an OFDM or base band system in comparison to the rectangular and raised cosine pulse modulated system and a BTRC modulated signal represents a better eye diagram with respect to that of the Rectangular and Raised cosine modulation [6].

**Case-IV: Proposed-I Pulse Shapes**

\[ G(f) = T, \text{ for } 0 \leq |f| \leq \frac{(1-\alpha)}{2T} \]

\[ G(f) = \frac{T}{2} \left[ (\beta -2\alpha -1)\beta \cos \left( \frac{\pi T}{\alpha} \left| \left| f \right| - \frac{1-\alpha}{2T} \right| \right) \right], \text{ for } \frac{(1-\alpha)}{2T} \leq |f| \leq \frac{(1+\alpha)}{2T} \]

\[ G(f) = 0, \text{ otherwise} \]

(7)

where, “\( \alpha \)” is roll off factor lies in between zero and one and “\( \beta \)” is the weighting coefficient of proposed-I such that \( 0 \leq \beta \leq 1 \).

**Case-V: Proposed-II Pulse Shape**

\[ G(f) = T, \text{ for } 0 \leq |f| \leq \frac{(1-\alpha)}{2T} \]

\[ G(f) = \frac{T}{2} \left[ \beta - 4\beta \alpha (1-\beta) \cos \left( \frac{\pi T}{\alpha} \left| \left| f \right| - \frac{1-\alpha}{2T} \right| \right) \right], \text{ for } \frac{(1-\alpha)}{2T} \leq |f| \leq \frac{(1+\alpha)}{2T} \]

\[ G(f) = 0, \text{ otherwise} \]

(8)

where, \( \beta \) (\( 0 \leq \beta \leq 1 \)) is the weighting coefficient of proposed-II pulse shape and “\( \alpha \)” is roll of factor of proposed-II pulse shapes.

These above said pulse shapes are expected to provide better BER performance than the other pulse shapes defined earlier in this section.

**Effect of Pulse Shaping on BER Performance in OFDM**

**Case-I: Effect of Rectangular Pulse Shaping on BER**

Considering the spectrum of a rectangular pulse from equation (4) and using equation (3) the second moment or power contained in this type of pulse can be evaluated as-
\[ E[\rho^2] \equiv \frac{1}{T} \int_{-\infty}^{\infty} |G(f)|^2 df \]
\[ E[\rho^2] = \frac{1}{T} \int_{-\pi}^{\pi} \frac{T^2}{\pi^2} \cdot \frac{\pi}{2\pi} \cdot \frac{T}{2} df \]
\[ E[\rho^2] = \frac{1}{T} \int_{-\pi}^{\pi} \frac{\pi}{2\pi} \cdot \frac{T}{2} df \]
\[ E[\rho^2] = \frac{1}{\pi} \]

Case-II: Effect of RC Pulse Shaping on BER
Consider the spectrum of RC pulse from equation (5) and using the equation (3) for evaluating the second moment as-
\[ E[\rho^2] = \frac{1}{T} \int_{-\infty}^{\infty} |G(f)|^2 df \]
\[ E[\rho^2] = \frac{1}{T} \left\{ 2T^2 \left(1 - \frac{\alpha}{2T}\right)^2 + \frac{T^2}{\pi^2} \int_{-\pi}^{\pi} \left[ 1 + \cos\left( \frac{\alpha T}{\alpha} \cdot f \right) \right]^2 df \right\} \]
\[ E[\rho^2] = (1 - \alpha) + \frac{\alpha}{2} \int_0^{\pi} \left( 1 + \cos(\pi \cdot u) \right)^2 du \]
\[ E[\rho^2] = (1 - \alpha) + \frac{\alpha}{2} \left[ 1 + \int_0^{\pi} \frac{1 + \cos(2\pi \cdot u)}{2} du \right] \]
\[ E[\rho^2] = (1 - \alpha) + \frac{\alpha}{2} \left[ 1 + \frac{1}{2} \right] = (1 - \alpha) + \frac{3\alpha}{4} \]
\[ E[\rho^2] = 1 - \frac{\alpha}{4} \]

Case-III: Effect of BTRC Pulse Shaping on BER
Consider the spectrum of BTRC pulse from equation (6) using the equation (3) for evaluation of second moment for this type of pulse as-
\[ E[\rho^2] = \frac{1}{T} \int_{-\infty}^{\infty} |G(f)|^2 df \]
\[ E[\rho^2] = \frac{1}{T} \left\{ 2T^2 \left(1 - \frac{\alpha}{2T}\right)^2 + \frac{T^2}{\pi^2} \int_{-\pi}^{\pi} \left[ 1 - \exp\left( \frac{2\alpha T}{\alpha} \cdot f \right) \right]^2 df \right\} \]
\[ E[\rho^2] = (1 + \alpha) + \frac{3\alpha}{8} \left( \frac{\alpha}{\log 2} \right) + \left( \frac{\alpha}{\log 2} - \frac{3\alpha}{8} \right) \]
\[ E[\rho^2] = 1 - \frac{\alpha}{4\log 2} \]

Case-IV: Effect of Proposed-I Pulse Shaping on BER
Consider the spectrum of Proposed-I pulse from equation (7) using the equation (3) for evaluation of second moment for this type of pulse as-
\[ E[\rho^2] = \frac{1}{T} \int_{-\infty}^{\infty} |G(f)|^2 df \]
\[ E[\rho^2] = \frac{1}{T} \left\{ 2T^2 \left(1 - \frac{\alpha}{2T}\right)^2 + \frac{T^2}{\pi^2} \int_{-\pi}^{\pi} \left[ 1 - \exp\left( \frac{2\alpha T}{\alpha} \cdot f \right) \right]^2 df \right\} \]
\[ E[\rho^2] = (1 + \alpha) + \frac{3\alpha}{8} \left( \frac{\alpha}{\log 2} \right) + \left( \frac{\alpha}{\log 2} - \frac{3\alpha}{8} \right) \]

at \( \beta = 1/2 \), above equation becomes as-
\[ E[\rho^2] = 1 - \frac{57\alpha}{64} \]

Case-V: Effect of Proposed-II Pulse Shaping on BER
Consider the spectrum of Proposed-II pulse from equation (8) using the equation (3) for evaluation of second moment for this type of pulse as-
\[ E[\rho^2] = \frac{1}{T} \int_{-\infty}^{\infty} |G(f)|^2 df \]
\[ E[\rho^2] = \frac{1}{T} \left\{ 2T^2 \left(1 - \frac{\alpha}{2T}\right)^2 + \frac{T^2}{\pi^2} \int_{-\pi}^{\pi} \left[ 1 - \exp\left( \frac{2\alpha T}{\alpha} \cdot f \right) \right]^2 df \right\} \]
\[ E[\rho^2] = (1 + \alpha) + \frac{3\alpha}{8} \left( \frac{\alpha}{\log 2} \right) + \left( \frac{\alpha}{\log 2} - \frac{3\alpha}{8} \right) \]

at \( \beta = 1/2 \), above equation becomes as-
The value of $E[\rho_i^2]$ for different types of pulse shapes in OFDM with QAM.

<table>
<thead>
<tr>
<th>Types of Pulse Shapes</th>
<th>Second Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular Pulse</td>
<td>$E_R[\rho_i^2] = \frac{1}{\pi}$</td>
</tr>
<tr>
<td>RC Pulse</td>
<td>$E_{RC}[\rho_i^2] = 1 - \frac{\alpha_{RC}}{4}$</td>
</tr>
<tr>
<td>BTRC Pulse</td>
<td>$E_{BTRC}[\rho_i^2] = 1 - \frac{\alpha_{BTRC}}{4\log 2}$</td>
</tr>
<tr>
<td>Proposed-I Pulse</td>
<td>$E_{Proposed-I}[\rho_i^2] = 1 - \frac{5\alpha_{Proposed-I}}{64}$</td>
</tr>
<tr>
<td>Proposed-II Pulse</td>
<td>$E_{Proposed-II}[\rho_i^2] = 1 - \frac{29\alpha_{Proposed-II}}{32}$</td>
</tr>
</tbody>
</table>

In order to maintain the same BER performance, it is mandatory that the equivalent noise power must be equivalent to each other, i.e.

$$E_{RC}[\rho_i^2] = E_{BTRC}[\rho_i^2] \text{ or } 1 - \frac{\alpha_{RC}}{4} = 1 - \frac{\alpha_{BTRC}}{4\log 2}$$

Hence, $\alpha_{BTRC} = 0.693\alpha_{RC}$

(14)

But, it is clear from equation (10) and (11) that BTRC pulse has lower equivalent noise power; therefore it will have better BER performance as compared to Rectangular and RC pulses.

The second moment of Interference “I” completely described the Gaussian distribution and its variance defined as:

$$Var[I] = \sigma_{ICI}^2 = E[I^2] = \frac{\Omega_s T \sum_{j=1}^L P_j E[\rho_i^2]}{4}$$

(15)

The value of $E[\rho_i^2]$ is derived in equations (9-13) for different types of pulse shapes in OFDM with QAM. The total equivalent noise power is defined as:

$$\sigma_i^2 = 1 + Var[I]$$

(16)

Hence, average BER, condition on $R_s$, is defined by:

$$P_b|_{R_{sr}} = Q \left\{ \frac{P_T T}{2\sigma_i^2}, r_s \right\}$$

$$\alpha \uparrow - E[\rho_i^2] \downarrow - Var[I] \downarrow - \sigma_i^2 \downarrow - P_b (BER) \downarrow \text{ eq. (15) to 19) - eq. (21)-eq. (22)-eq. (23)}$$

(17)

(18)

(19)

**Conclusion**

It is observed that the BER performance obtained from the Gaussian Approximation depends only on total interference signal power as per equation (15). It is also observed that larger value of $\alpha$ gives smaller value for mean square value $\{E[\rho_i^2]\}$ and hence smaller values for $Var[I]$ and $\sigma_i^2$, as from equation (17), it is clear that the BER will also be correspondingly reduced. Therefore, from the Gaussian Approximation equation (17), it is clear that the value of excess bandwidth $\alpha$ provides a tradeoff between spectral efficiency and detection performance in an OFDM system.

Comparing the performance of Rectangular, Raised cosine and BTRC form equation (9) to (11), the BTRC gives smaller equivalent noise power and hence lower BER values. Therefore, the BTRC pulse is supposed to be a better pulse shaping for OFDM in terms of obtaining better BER performance [6].

It can be concluded that the two proposed pulses are giving better $E[\rho_i^2]$ in respective cases which in turn provides better BER as compared to BTRC which was considered as one of the best pulse shape [6].

**References**


Fig. 1: RC and BTRC plotted in time domain for different value of $\alpha$.

Fig. 2: Time domain of Proposed-I Pulse for different value of $\beta = 0$, $\beta = 0.5$ & $\beta = 1$.

Fig. 3: Times domain Representation of Proposed-II Pulse Shape for different value of $\beta$. 