Magic Number in Neutron-Rich Nuclei using Relativistic Mean Field Formulism

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Abstract
The magic numbers associated with closed shells have long been assumed to be valid across the whole nuclear chart. Investigations in recent years of nuclei in the drip-lines have revealed that the magic numbers may change locally in the exotic nuclei leading to the disappearance of shell gaps and the appearance of new magic numbers. We investigate the magic number in neutron drip-line using axially deformed Relativistic Mean field Theory. The neutron numbers N=28, in $^{52}$Ca and N=40 in $^{52}$Ca become magic or semi magic, while its magicity is broken in $^{60}$Ni. A considerable shell gap at N=40 appears in $^{68}$Ni and $^{70}$Cabut almost disappears in $^{86}$Ni.

Index Terms: Nuclei, magic number, binding energy, drip-line, single particle energy, quadrupole deformation, separation energy.

Introduction
The magic number is the backbone of nuclear structure physics that explains the structure of nuclei close to stability. The magic number is the backbone of nuclear structure. The origin of magic number is in the gaps created by the single-particle eigen states of the mean-field. Exploring the formation of shell structure away from the $\beta$-stability line. The new generation of RIBF will attempt give the possible answer to one of the questions that what is the shell configuration of nuclei at the extremes of iso-spin. The answers to the questions have started emerging, for example, exotic nuclei $^{4}\text{Si}$ [1] and $^{78}\text{Ni}$ [2] have been produced at the National Superconducting Cyclotron of Michigan State University. The first evidence for the N = 16 magic number in oxygen came from an evaluation of neutron separation energies on the basis of measured masses [7]. The measurements showed some surprising changes in the nuclear shell structure as a function of proton and neutron number in light nuclei. These observations triggered numerous theoretical investigations, which in turn made new predictions that some magic numbers will disappear and new shell gaps will appear in certain regions of the nuclear chart [3]. The disappearance of the magic numbers N = 8 and 20 in the light nuclei, and/or appearance of the new magic numbers N = 16 and 32 in drip-line nuclei have been reported recently and can be found in Refs. [4-7]. One case of particular interest is the magicity at proton/ neutron number N = Z = 28 and N = 32 and 40 near the neutron drip-line in Ca and Ni-isotopes, which has been a centre of discussion for sometimes now (see Ref.[8], and the references therein). Recently, N=40 is reported to be magic in HF calculations with a number of effective interactions [9]. In the present investigation we calculate the single particle energy levels, quadrupole deformation parameter $\beta$ and separation energy using axially deformed relativistic mean field model. A brief description of model and the analysis of the result are presented in the following sections.

Lagrangian Density
The RMF model has been proved to be a very powerful tool to explain the properties of finite nuclei and infinite nuclear matter [10], [11], [12] for the last two decades. We start with the relativistic Lagrangian density for a nucleon-meson many-body system,

$$
L = \bar{\psi}_i \left( i \gamma_{\mu} \partial_{\mu} - M \right) \psi_i + \frac{1}{2} \partial_{\mu} \sigma \partial^{\mu} \sigma - \frac{1}{2} m_{\sigma}^{2} \sigma^{2} - \frac{1}{3} g_{2} \sigma^{3} - \frac{1}{4} g_{3} \sigma^{4} - g_{4} \bar{\psi}_i \gamma_{\mu} \gamma_{5} \psi_i \sigma - \frac{1}{4} \Omega^{\mu\nu} \Omega_{\mu\nu} + \frac{1}{2} m_{\omega}^{2} \psi^{\mu} \psi_{\mu} + \frac{1}{4} c_{4}(\nu_{\mu} \nu^{\mu})^{2} - g_{\omega} \bar{\psi}_i \gamma^{\mu} \psi_i \nu_{\mu} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - e \bar{\psi}_i \gamma^{\mu} \frac{(1 - \gamma_{5})}{2} \nu_{\mu} \psi_i A_{\mu} 
$$

(1)

The field for the $\sigma$-meson is denoted by $\sigma$, that for the $\omega$-meson by $\nu_{\mu}$ and for the isoo-vector $\rho$-meson by $\tilde{R}_{\mu}$. $A_{\mu}$ denotes the electromagnetic field. The $\psi_{i}$ are the Dirac spinors for the nucleons whose third component of isospin is denoted by $t_{3i}$. Here $g_{\sigma}$, $g_{\omega}$, $g_{\rho}$ and $c_{4}$ are the coupling constants for $\sigma$, $\omega$, $\rho$ mesons and photon, respectively. $g_{2}$, $g_{3}$ and $c_{4}$ are the parameters for the nonlinear terms of $\sigma$- and $\omega$-mesons. $M$ is the mass of the nucleon and $m_{\sigma}$, $m_{\omega}$ and $m_{\rho}$ are the masses of the $\sigma$, $\omega$ and $\rho$-mesons, respectively. $\Omega^{\mu\nu}, \tilde{R}_{\mu}$ and $F^{\mu\nu}$ are the field tensor for the $\nu_{\mu}$, $\tilde{R}_{\mu}$ and the photon fields, respectively[13].

From the relativistic Lagrangian we get the field equations for the nucleons and mesons. These equations are solved by expanding the upper and lower components of Dirac spinors and the Boson fields in a deformed harmonic oscillator basis with an initial deformation. The set of coupled equations is solved numerically by a self-consistent iteration method. The centre-of-mass motion is estimated by the usual harmonic oscillator formula $E_{c.m.} = \frac{1}{2} (41A^{-1/3})$. The quadrupole deformation parameter $\beta_{2}$ is evaluated from the resulting
quadrupole moment \([13]\) using the formula,
\[ Q = Q_n + Q_p = \sqrt{\frac{2}{5\pi}} AR^2 \beta_z, \]

(2)

Where, \( R = 1.2A^{1/3} \). The total binding energy of the system is,
\[ E_{\text{Total}} = E_{\text{Part}} + E_\sigma + E_\omega + E_p + E_c + E_{\text{pair}} + E_{\text{cm}} \]

(3)

where \( E_{\text{Part}} \) is the sum of the single-particle energies of the nucleons and \( E_\sigma, E_\omega, E_p, E_c \) and \( E_{\text{pair}} \) are the contributions of the mesons fields, the Coulomb field and the pairing energy, respectively. For the open shell nuclei, the effect of pairing interactions is added in the BCS formalism. For pairing strength, the BCS approach provides a reasonably good description of nuclei close to \( \alpha \) not too far away from stability line. For the nuclei in the vicinity of drip-lines, coupling to continuum becomes important. However, it has been shown that self-consistent treatment of BCS approximation breaks down when coupling between bound states and the states in continuum takes place \([14]\). The pairing gaps for proton \( \Delta_p \) and neutron \( \Delta_n \) are calculated from the relations \([15]\),
\[ \Delta_p = r b_p Z^{-1/s} e^{(s-l+t^2)} \]
\[ \Delta_n = r b_n N^{-1/s} e^{-(s+l+t^2)} \]

(4)

where \( r = 5.72\text{MeV} \), \( s = 0.118 \), \( t = 8.12 \), \( b_p = 1 \) and \( I = (N-Z)/(N+Z) \).

**Results and Discussion**

In the present calculations we use the axially deformed relativistic mean field (RMF) Theory with NL3 parameter set. The pairing interaction is taken care within the BCS scheme, with gap parameters as in Ref. \([16]\).

The results of calculations for the neutron single particle energy levels for \( ^{60,68,78}\text{Ni} \) and \( ^{52,60}\text{Ca} \) nuclei are, respectively, shown in Figs. 1 and 2. In figure 1, the shell gap of \(-4.0\text{MeV} \) can be seen at \( N = 28 \) and \( N = 40 \) for all the nuclei considered here. Although, the gaps are not very large but fairly clear at these numbers. The two neutron separation energy is shown in Fig. 3. The magicity corresponding to \( N = 28 \) and 40 can easily be found at sudden fall in the separation energies of \( ^{52-80}\text{Ni} \) and \( ^{40-76}\text{Ca} \) nuclei. In case of Ni, the sharp decrease in separation energy at \( N = 50 \) is obtained, which means the magicity at \( N = 50 \) is also indicated in RMF calculations. The sharp decrease in the separation energy at \( N = 28 \) and 40 is in agreement with the single particle energy levels in Fig. 1, which further gives the insight of the magicity. Considerably large shell gaps (\( \approx 6\text{MeV} \)), both at \( N = 28 \) and 40 in \( ^{68}\text{Ni} \) nucleus appears. Note that this nucleus has been suggested to be a doubly magic nucleus experimentally \([17]\). Similarly, in \( ^{78}\text{Ni} \) nucleus, the large shell gaps at \( N = 28, 40 \) and 50 are obtained. This shows that the magicity at \( N = 28 \) is perhaps not diminished, and the gap at \( N = 50 \) is nearly \(-4.0\text{MeV} \). Fig. 5, showing a plot of neutron matter radius for \( ^{40-76}\text{Ca} \) isotopes, in which a sharp change in radius at \( N = 28 \) and 40. The considerably large difference in the radii at \( N = 28 \) and 40 gives the indication of the extra stability of nuclei at these numbers. In Fig. 4, we present the variation of the pairing gap \( (\Delta_n) \) for neutron for the different paring strengths. The pairing gap vanishes at \( N = 28 \) and 40 at small pairing strength of \( G_n = G_p = 0.1 \) showing the magicity at these numbers.

Fig. 1. The neutron single particle energy levels of \(^{60,68,78}\text{Ni} \) nuclear with NL3 parameter set.

Fig. 2. The neutron and proton single particle energy levels of \(^{52,60}\text{Ca} \) nuclei with NL3 parameter set.

Fig. 3. Two neutron separation energy of Ca and Ni isotopes with NL3 parameter set.
Fig. 4. The variation of pairing gap of $^{38-80}$ Ca isotopes for different strength with NL3 parameter set.

Fig. 5. The neutron radius of $^{38-68}$ Ca nuclei as a function of neutron number using NL3 parameter set.

Conclusion

We have investigated shell structure of the neutron-rich Ca and Ni nuclei using axially deformed Relativistic Mean field Model with NL3 parameter set. At $N = 32$ a small gap in single particle energy levels is obtained which cannot account for the magic nature, but shell-model calculations using the well established KB3G interaction [18],[19] support a $N = 32$ shell closure, which is experimentally well established, but not a $N = 34$ shell closure, whereas the gaps at $N = 28$ and 40 are considerably large in both the cases considered here. The same is seen in the two neutron separation energy also. The present calculations do not support the $N = 34$ magic number. Therefore, in RMF calculations with NL3 parameter set show the magicity at $N = 28$, 40 and 50 for the isotopes of Ca and Ni.

References